Toward Autonomy Realism

from Chapter 5 of *Numbers without Science*

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Revised: Summer 2010



Let's call mathematical realism the position that a) mathematical objects exist; and/or b) some mathematical claims are true, but not vacuously so. The most promising defense of mathematical realism in the last fifty years has used some form of indispensability argument on which our knowledge of mathematics depends on our knowledge of empirical science. Paradigmatically, we can consider Quine's indispensability argument, which I'll call QI.

QI
- QI.1: We should believe the theory which best accounts for our empirical experience.
- QI.2: If we believe a theory, we must believe in its ontic commitments.
- QI.3: The ontic commitments of any theory are the objects over which that theory first-order quantifies.
- QI.4: The theory which best accounts for our empirical experience quantifies over mathematical objects.
- QI.C: We should believe that mathematical objects exist.[1]

Lamentably, for the defender of the indispensability argument, the resultant mathematical realism is strictly non-traditional. While the indispensability argument does justify belief in some mathematical theorems, the objects which satisfy those theorems need not be mathematical objects as ordinarily conceived. In particular, the indispensability argument entails certain unfortunate consequences.

*UC.1: Restriction*: The indispensabilist's commitments are to only those mathematical objects required by empirical science.
*UC.2: Ontic Blur*: The indispensabilist's mathematical objects are concrete.
*UC.3: Modal Uniformity*: The indispensabilist's mathematical objects do not exist necessarily.

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[1] See Quines 1939, 1948, 1951, 1955, 1958, 1960, 1978, and 1986. Other versions of the indispensability argument are available. See Putnam 1971 (the success argument); Resnik 1997: §3.3 (the pragmatic indispensability argument); and Mancosu 2008: §3.2 (the explanatory indispensability argument) for a few examples.

*UC.4: Temporality*: The indispensabilist's mathematical objects exist in time.
*UC.5: Aposteriority*: The indispensabilist's mathematical objects are known a posteriori.
*UC.6: Methodological Subservience*: Any debate over the existence of a mathematical object will be resolved, for the indispensabilist, by the needs of empirical theory.[2]

These failures of the indispensability argument to establish a robust mathematical realism have been widely discussed, if not accepted, and I shall not pursue them here. In this paper, I sketch the trail that a mathematical realist must blaze in light of the failure of the indispensability argument. I use 'autonomy realism' for a position on which a) mathematical objects exist; b) some mathematical claims are true, but not vacuously so; and c) our knowledge of these objects and truths does not depend on our knowledge of empirical science. I defend the pursuit of autonomy realism.

My argument, distilled, is:

AR
AR.1: We have mathematical knowledge.
AR.2: Our mathematical knowledge must either be strictly derived from our scientific theories, or it must be autonomous.
AR.3: Our mathematical knowledge is not strictly derived from our scientific theories.
AR.C: Thus, autonomy realism.

Besides avoiding the Unfortunate Consequences that beset the indispensability argument, autonomy realism is motivated by four considerations which I will briefly note. First, mathematical posits are different kinds of posits than those of empirical objects, including odd empirical objects like fields and space-time points. They arise differently, and they are tested differently. They are governed by axioms which are independent of those of physical science. Second, autonomy realism allows a semantics which treats mathematical sentences generally according to their surface grammar.

Third, autonomy realists approve of Euclidean rescues, on which changes in scientific theory do not tell against mathematical theories.[3] When relativity supplanted classical mechanics, the

[2] See my "Why the Indispensability Argument Does Not Justify Belief in Mathematical Objects."

[3] The term 'Euclidean rescue' is due to Michael Resnik. See Resnik 1997: 130.

structure of space-time was discovered to be non-Euclidean. A Euclidean rescue defends both the new mathematical theory and the old one despite the change in physical theory. All three possibilities concerning the parallel postulate are consistent with unfalsified geometric theories.[4] Euclidean rescues are useful for a variety of mathematical puzzles, including how to understand the status of the continuum hypothesis, which I discuss below, in §2.1. We can perform a Euclidean rescue any time a mathematical theory fails to apply in science. In such cases, the indispensabilist must reject the mathematics, or demote it to recreation due to a lack of empirical evidence. The traditional response, open to the autonomy realist, is the Euclidean rescue, unless the mathematics is shown inconsistent.

A fourth motivation for autonomy realism comes from the indeterminacy problems like that discussed by Paul Benacerraf in "What Numbers Cannot Be." Benacerraf's problem was that different sets of sets can model the Peano axioms, and it appears that nothing can tell us which one of these sets is the right one. Autonomy realism can support a *sui generis* solution to Benacerraf's problem on which numbers are not reduced to sets at all; numbers are numbers. The *sui generis* solution avoids the problem of multiple set-theoretic models of the Peano axioms by denying that we should expect any set-theoretic model to be uniquely correct. Number-theoretic axioms are modeled by the numbers, and there are various set-theoretic dopplegangers.

Autonomy realism supports the *sui generis* solution, since it allows for diverse

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[4] Euclid's parallel postulate states that if a line intersects two other lines and makes the interior angles on the same side less than two right angles, then the two lines meet on that side. The parallel postulate is equivalent to Playfair's Postulate, which states that given a line and a point not on that line, exactly one line can be drawn through the given point parallel to the given line. There are two ways to deny Playfair's postulate, or the parallel postulate, both of which are consistent with the other axioms of geometry. If one can draw no parallel lines, the geometry defines the surface of a sphere. If one can draw more than one parallel line, one defines a surface called a hyperbolic spheroid, or a pseudo-sphere.

mathematical objects. Robust mathematical intuitions can support set theory, number theory, geometry, and more specific mathematical theories, without necessarily reducing them to one unifying mathematical theory. We need not see the translations between numbers and sets as reductive. One can view such translations as arguments for the acceptance of non-sets. If we are convinced of the consistency of set theory, and we note that the objects of some other mathematical structure are translatable to sets, then that is good argument for the consistency of the non-set-theoretic structure, and thus good reason to believe in the additional elements, too. Translations work in two directions.

This paper consists of three parts. In Part 1, I argue that mathematics shares important characteristics with empirical science, characteristics which confer legitimacy on an autonomous mathematical methodology. In Part 2, I characterize autonomy realism. In Part 3, I show that autonomy realism is preferable to a strong version of anti-realism, Hartry Field's fictionalism.

## Part I: The Legitimacy of Mathematics

### §1.1: The Limits of Logic as Mathematics

The unfortunate consequences of the indispensability argument do not extend to our knowledge of many logical theories, especially those, like standard first-order logic, which make no ontological commitments. Even weak logical theories, though, afford the formal theorist some tools which might ordinarily be thought to require mathematics, and we need logic to govern inferences in scientific theory anyway. I begin Part 1 by exploring the mathematical tools that logic provides in the absence of an indispensability argument. Finding these insufficient, I proceed to argue for the legitimacy of mathematics in its own right.

The extent of the mathematics attainable from logic depends on which logic one takes as canonical. Propositional logic provides few mathematical resources. From first-order logic, if we include an identity predicate and its standard governing axioms, we can generate logical dopplegangers for finite natural numbers. Specifically, we can generate finite cardinality quantifiers, allowing us to distinguish the sizes of different finite collections. We can say that there are three, four, or seventeen thousand blue vases in the showroom, or planets in the galaxy.

The would-be indispensabilist also has concrete templates at his disposal. Templates represent other concrete patterns, and can perform practical geometric tasks, by serving as blueprints and allowing us to measure and design spaces.[5] But concrete templates and cardinality quantifiers are insufficient if scientific theory requires even the full theory of natural numbers, let alone analysis. We can not represent continuity or infinite sizes. But it is generally accepted that scientific theories of space-time require continuity, and perhaps infinity.

One might adopt a stronger logic. Hartry Field has explored a logic of plural quantification, one which is equivalent to a mereological theory, to handle collections of space-time points and regions.[6] Stewart Shapiro recommends second-order logic, which yields rudimentary set theory. With any logic which generates some sort of set theory, one can construct objects which function like the natural numbers, and other mathematical objects.

Still, the claim that we can avoid mathematical commitments on the basis of a strong logic rings hollow. Field's logic was criticized for being too mathematical; mereological axioms entail theorems too substantial to be called logical.[7] We can see the objects generated by mereological theorems and second-order logic as replacements for sets and other mathematical objects only because we already possess profound information about mathematics. We take '{{φ}}' as a two, for example, on the basis of a

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[5] See Resnik 1997: 226 ff.

[6] See Field 1980: Chapter 9; and Field 1989: Chapter 3.

[7] See Malament 1982, Shapiro 1983, and Field 1984a: 141. More generally, Quine 1986 famously derided second-order logic as set theory in sheep's clothing.

translation between the objects intended as the models of the Peano axioms and a set-theoretic sequence. We can only take sets to serve the functions of numbers if they provably perform all the tasks that numbers do. Similarly, we can only take the objects of second-order logic as sets if they do the work of sets. Our prior knowledge of sets, or of numbers, is a constraint on the claim that we can substitute logical theories for mathematical ones. Anyone who attempts to restrict her ontology by adopting stronger logics would disingenuously pretend to eliminate mathematics while using mathematical knowledge as a constraint on the adoption of a logic and the consequent construction of scientific theory.

Focusing on the logic used in science is instructive, though, for it shows how the appeal to scientific theory in even an ideal indispensability argument is insufficient to generate mathematical ontology. Consider a giant book in which is inscribed the spatio-temporal position of every object in the universe at every moment. This could be done with no mathematics, since we can use the dopplegangers constructible out of first-order logic. If one wanted to predict the position of any object at any time, say, or the direction of its motion, all you would have to do is look it up. Such a book would perform just about all the functions of scientific theory that we could want. It would be perfectly precise and perfectly predictive, if inelegant. It would need no mathematics beyond that generated by first-order logic. If our knowledge of mathematics truly depended on its indispensable utility for empirical science, we seem unable to justify any mathematical knowledge at all.

Furthermore, accounts of our knowledge of logic typically appeal to mathematical theories. We use set theory to model first-order logic, and any logic strong enough to do the work for science. So, even appeals to an ontologically uncontroversial logic seem to entail commitments to mathematics.

The mathematical realist should resist reliance on logical substitutes for mathematics. Any strong logic hides mathematical claims, and requires mathematics for its interpretation. Scientists need more than first-order logical machinery, whether in the guise of mereological axioms which govern the structure of a substantivalist space-time, or in the guise of traditional mathematical theories.

§1.2: Bootstrapping

In the absence of scientific justifications of mathematical knowledge, given the Unfortunate Consequence of the indispensability argument, one might wonder whether mathematics can justify itself. Perhaps we should admit mathematical objects into our ontology, not because of their indispensable use in physics or biology or psychology or economics, but because they are the objects to which mathematical theories themselves refer. Similar arguments could be made about a variety of special sciences. We should admit the objects of biology (e.g. DNA sequences) because of their indispensable uses in biology. We should admit the existence of preference orderings because of their uses in economics. If we accept the legitimacy of mathematics in its own right, the indispensability argument is superfluous.

Unfortunately, such a possibility appears to be rhetorically unsound bootstrapping. By parity of reasoning, we could justify admitting the existence of ghosts or witches as being required for the study of paranormal phenomena. The argument for the existence of the objects of a discipline can not depend solely on their indispensability within that field. We must also establish the legitimacy of the discipline itself. The purported science of paranormal phenomena, unlike biology, does not have the legitimacy of a proper science. In order to know whether mathematics is a proper science, we need criteria which determine whether a discipline is legitimate. In the philosophy of science, the problem of settling on these criteria is known as the demarcation problem. Without a solution to the demarcation problem, or at least a demonstration that any solution must pronounce mathematics legitimate, any claim for the legitimacy of mathematics is just bootstrapping.

Still, Penelope Maddy and Hilary Putnam have each flirted with positions which fall to a bootstrapping criticism, and I discuss their proposals with an eye to avoiding the problems that they raise.

§1.2.1: Maddy and Bootstrapping

In an attempt to justify our knowledge of pure mathematics, Maddy 1992 modifies and extends Quine's indispensability argument. She wants to avoid UC.1, Restriction, by claiming that mathematical practice, rather than empirical science, should determine mathematical ontology, and that practicing mathematicians believe in a fuller mathematical universe than empirical science requires.

Maddy's modified indispensability argument first appeals to the indispensable applications of mathematics to convince us generally that there are mathematical objects. She does not specify which version of the indispensability argument she intends; I presume she means some version of Quine's QI. Then, Maddy notes that the appeal to QI is inadequate to justify mathematical practice.

> [I]ndispensability theory cannot account for mathematics as it is actually done... [W]e must conclude that the indispensability arguments do not provide a satisfactory approach to the ontology or the epistemology of mathematics. Given the prominence of indispensability considerations in current discussions, this would amount to a significant reorientation in contemporary philosophy of mathematics (Maddy 1992: 289).

As part of this reorientation, Maddy appeals to mathematical practice to justify specific results, including the more abstruse, pure results.[8]

> A modified indispensability argument first guarantees that mathematics has a proper ontology, then endorses (in a tentative, naturalistic spirit) its actual methods for investigating that ontology. For example, the calculus is indispensable in physics; the set-theoretic continuum provides our best account of the calculus; indispensability thus justifies our belief in the set-theoretic continuum, and so in the set-theoretic methods that generate it; examined and extended in mathematically justifiable ways, this yields Zermelo-Fraenkel set theory (Maddy 1992: 280.)

Extending QI as Maddy suggests seems to conflict with Quine's claim that all of our ontological commitments are determined in the same way, by looking at the domain of quantification of our best scientific theory. But Maddy does not take herself to be abandoning Quine's naturalism for an autonomy realism. Instead, she believes that a proper naturalism allows for the inclusion of pure mathematical axioms in our best theory, even if they are not needed for empirical science.

As Maddy notes, there are two different interpretations of 'naturalism' which might be relevant.

[8] Maddy may have taken her cue from remarks in Field 1980: 4, where Field suggests that the indispensabilist can have more than just a little rounding-out beyond the results needed to account for application.

On a narrow interpretation, we only add mathematical axioms as they are required by empirical science; this is the indispensabilist's position. Maddy's broader naturalism takes mathematics itself to be naturalistically defensible. The broader version justifies mathematical knowledge directly, since we just add the axioms yielded by our inde-pendent mathematical methodology to our best theory. Maddy's naturalism thus renders the indispensability argument otiose.

By itself, Maddy's broader naturalism provides an incomplete justification of pure mathematics. We must ask why the naturalist should accept mathematics as legitimate, since any practice can not justify itself. Maddy writes, "Mathematics, after all, is an immensely successful enterprise in its own right, older, in fact, than experimental natural science. As such, it surely deserves a philosophical effort to understand it as practiced, as a going concern... [A] philosophical account of mathematics must not disregard the evidential relations of practice or recommend reforms on non-mathematical grounds" (Maddy 1992: 276).

If Maddy's justification of pure mathematics comes only from mathematical practice, by parity of reasoning the psychic's reliance on the crystal ball and divine revelation can also be justified. Appeals to psychic powers and revelation are also older than experimental natural science. While mathematical practice may be successful explaining mathematical phenomena, psychic powers may be successful in explaining psychic phenomena. The natural scientist claims that ghosts do not exist, and thus we need no account of our knowledge of them. Similarly, the scientist with nominalist tendencies claims that mathematical objects do not exist, and thus we need no account of them.

Even if we set aside this bootstrapping problem, Maddy's modification does nothing to solve the problems of the other Unfortunate Consequences. For example, the mathematics which results from Maddy's argument, like that which results from any indispensability argument, suffers from UC.6, Uniqueness. Various answers to the continuum hypothesis lead to various incompatible, though provably consistent, set theories. But, mathematical practice allows us to accept all of the competing answers, perhaps as competing results of independent axiomatizations. The indispensabilist is committed only to those which apply to the physical world. Maddy is forced to accommodate two irreconcilable views.

More importantly, Maddy does not provide sufficient criteria to distinguish mathematics, as a legitimate discipline, from illegitimate pursuits. She merely says that it is a longstanding pursuit successful on its own terms. Putnam makes a similar mistake.

§1.2.2: Putnam and Bootstrapping

Putnam presents an argument for mathematical realism, called the success argument, which is open to several different interpretations.

> I believe that the positive argument for realism [in science] has an analogue in the case of mathematical realism. Here too, I believe, realism is the only philosophy that doesn't make the success of the science a miracle (Putnam 1975a: 73).

Putnam's success argument might merely be taken as an indispensability argument.

MSI
MS1. Mathematics succeeds as the language of science.
MS2. There must be a reason for the success of mathematics as the language of science.
MS3. No positions other than realism in mathematics provide a reason.
MSC. So, realism in mathematics must be correct.

MSI in turn may be seen as either an expression of QI or a modification of it. I will not pursue either of these interpretations.[9] Another interpretation of Putnam's success argument bootstraps.

MSB
MSB1. Mathematics succeeds in itself. That is, it is fruitful.
MSB2. There must be a reason for the success of mathematics.
MSB3. No positions other than realism in mathematics provide a reason.
MSBC. So, realism in mathematics must be correct.

Like Maddy's modified argument, MSB is insufficient to generate a justification of mathematics. We need antecedent criteria to determine whether a theory is successful in the sense of MSB1. If the

[9] But, see my "E Pluribus Putnams Unum Putnam," unpublished.

criteria come from within the theory, then any theory can be deemed successful. The crystal ball can tell you to believe the crystal ball.

Within mathematics, mathematical criteria reign. We can justify a posit, for example, by its fruitfulness. Consider Descartes's posit at the foundation of analysis.

AG There is a one-one correspondence between the points on a line and the real numbers.

Even lacking proof that there are as many points on a line as there are real numbers, the fruitfulness of analysis makes it indispensable for the practice of mathematics. Thus we should believe AG. This kind of indispensability is an inference to the best explanation and it yields justifications of particular mathematical theorems in the same way that a theoretical posit in physics yields electrons. But we can only entertain a statement such as AG if we have prior commitments to points and lines and real numbers. Inferences to the best explanation within a discipline are often essential to scientific and mathematical methods. We use them to justify posits of subvisible particles and fields in physical science, for example. They can not justify knowledge of an entire discipline like mathematics.

Maddy's modified indispensability argument and Putnam's MSB both suffer bootstrapping problems which can be solved if we find a reasonable solution to the demarcation problem. Criteria for good science should rule out obviously unacceptable fields, like parapsychology, and rule in obviously acceptable ones, like empirical science. Then, we can see what they say about mathematics.

§1.3: Demarcation

Exact characterization of acceptable scientific methodology is a notoriously intractable problem. Still, there are some relatively uncontroversial claims. Good science produces replicable results. These results cohere with other accepted results. The methods used in good science receive broad acceptance. When these factors are absent, empirical results are dubious.

Burgess and Rosen present a detailed list of theoretical virtues which may be taken as a significant step to solving the demarcation problem.

D1. Correctness and accuracy of observable prediction.
D2. Precision of those predictions and breadth of the range of phenomena for which such predictions are forthcoming, or more generally, of interesting questions for which answers are forthcoming.
D3. Internal rigour and consistency or coherence.
D4. Minimality or economy of assumptions in various respects.
D5. Consistency or coherence with familiar, established theories.
D6. Perspicuity of the basic notions and assumptions.
D7. Fruitfulness, or capacity for being extended to answer new questions (Burgess and Rosen 1997: 209).

This list need not be taken as a revolutionary insight. These criteria are just the ordinary constraints on scientists in their daily work. Neither need we take this list as a categorical solution to the demarcation problem. It only needs to be a good working hypothesis, and it shows how mathematics should be classified with the good sciences.

Merely by omitting 'observable' from D1, or by interpreting that word to apply to our observations of mathematical results, like a token of a proof, or to intuitions of mathematical truths, the list is perfectly applicable to mathematics. Mathematical theorems must be perspicuous, as in D6, and proofs, or at least proof methods, must be available for scrutiny and receive broad acceptance, as in D5. Also as in D5, mathematical results must cohere, especially results which bridge mathematical sub-fields. Consider the mathematical virtues of Wiles's proof of Fermat's theorem, which bridged topology and number theory. Wiles's proof increased the range of mathematical phenomena which topology predicts, as in D2. We seek alternative proofs of a theorem, which helps with D1, D3, and D5. A mathematical theory must be consistent, as in D3.[10] Axioms should be fruitful, as in D7, and few, as in D4. The consequences of axioms should be intuitively acceptable.

The weights we ascribe to the different factors may differ from those we ascribe in any particular

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[10]At least, it must be globally consistent. Dialetheic systems might contain local inconsistencies, but these have to be somehow isolated from the rest of a theory.

empirical case. Mathematicians emphasize consistency, D3; empirical scientists prefer to emphasize economy, D5. But empirical cases will differ amongst themselves, too. Physicists tend to weight precision, D2, more than psychologists, whereas the reverse seems to be true of perspicuity, D6.

The revised interpretation of observation in D1 will seem like a major concession to those who deny mathematical intuition. They will argue that observation is essential for validating our beliefs, whereas mathematical intuition will seem like magic.[11] But, the other criteria overwhelmingly favor considering mathematics as a science. We should not stack the deck in favor of one criterion alone. Weighting observable predictions over mathematical predictions, and over the other criteria, renders D2-D7 moot. Such a move would be acceptable only to someone with independent objections to taking mathematics to be a legitimate science. In that case, we are using the demarcation criteria, and really only D1, just to rule out mathematics.

One should not worry that a wider interpretation of D1 will actually grant legitimacy to pseudo-sciences. Pseudo-sciences like parapsychology fail to satisfy multiple criteria. Psychic practice, for example, posits conduits to knowledge which conflict with our best scientific theory while attempting to explain the same phenomena. Empirical evidence weighs heavily against such conduits. The psychic can say whatever he wants and his results are not replicable. They fail to cohere with accepted science. The psychic's methods are suspect at all of D1-D7.

The application of D1 - D7 to mathematics varies from their application to science, but they are the same broad, naturalist criteria. Applied to mathematics, the specific constraints on theory construction are not empirical. The phenomena explained by mathematics are mathematical facts, not empirical ones. Still, the list shows how mathematics works like good science.

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[11] Michael Devitt made this argument to me, in personal communication.

§1.4: Starting with Mathematical Objects

D1 emphasizes the predictiveness of good science. Mathematics, too, is predictive, of mathematical phenomena, of the behavior of mathematical objects.[12] If we accept mathematical methodology, we must also accept mathematical phenomena. Quine, and all indispensabilists, reject pure mathematical phenomena, though autonomy realists accept them. In this section, I argue that Quine adopts a kind of unacceptable foundationalism to reject autonomy realism.

We do not possess a complete account of A) what exists, and B) how we know what exists. A healthy attitude toward our speculation would be to seek an equilibrium between our best estimates of each. Quine's foundationalism consists of the insistence that we settle B according to strict empiricist tenets, and ensure that all claims A conform. QI presumes this method at QI.1. Quine's foundationalism does not pretend to establish certainty, but it does claim the epistemic cleanliness sought by traditional foundationalists. By leading with his empiricism, Quine maintains a latent positivist tendency to denigrate metaphysics, not by eliminating it, but by making it subservient to epistemology.

We can see Quine's foundational tendencies in his attitudes toward meanings, or intensions. In linguistics as in mathematics, Quine leads with his empiricism to deny the existence of intensional objects. If we admit sense properties, as Jerrold Katz argues we should, we can solve the supposed problems of indeterminacy of translation. Indeterminacy would be blocked by facts about meanings. But, the only meaning facts that Quine accepts are extensional, those which account for observable behavior. "As a general argument against the determinacy of translation, Quine's argument fails because it rules out evidence about sense properties and relations by fiat. It begs the question against the intensionalist who does not understand intension extensionally..." (Katz 1998: 90.)

Beliefs about sense properties, like mathematical beliefs, are beliefs of spatio-temporal beings.

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[12] Reference to the behavior of mathematical objects is language borrowed from observation, and is merely metaphoric. It is common in mathematics where one naturally talks of the behavior of functions, or how sets act under operations.

The best account of these beliefs may posit non-empirical justification. Such justifications may be acceptable to the naturalist as long as they are consistent with our epistemic capacities. Naturalism in this broad sense could account for mathematical facts.

There is thus a tension between Quine's naturalism and QI. QI relegates existence claims to the domain of quantification of our best empirical theory, and opposes autonomy realism, while naturalism in the broader sense can support an autonomous mathematics. "[I]f the nominalistic reformer is to claim to be an adherent of naturalization or naturalism in epistemology, the 'naturalism' in question must be of a restricted variety, making invidious distinctions, marginalizing some sciences (the mathematical) and privileging others (the empirical)" (Burgess and Rosen 1997: 211.)

Given criteria which justify mathematics as autonomous from empirical science, we need an account of our mathematical knowledge. Part of this account just refers to ordinary mathematical methodology. Another part parallels Quine's account of the construction of empirical theory. On Quine's account, the naturalist starts with ordinary objects and constructs a theory (or theories) to account for his experience. The autonomy realist may also start with mathematical objects and construct a theory (or theories) to account for mathematical phenomena. The question for the philosopher of mathematics, as for the philosopher of science generally, is to explain, "How it is that man works up his command of that science from the limited impingements that are available to his sensory surfaces" (Quine 1974: 3.)

Quine avoids sense-data reductionism in part due to the practical impossibility of a sense-data construction, but also on principle. The empiricist wanted unmediated, unassailable data as a starting point. But the notion of a sense datum is itself the product of a substantial theory about human perception and the way in which we gather information. Quine thus accepts our beliefs ordinary physical objects as defeasible starting points for the construction of scientific theories. Those theories are to be judged both on the basis of their ability to account for this initial evidence, and, since evidence under-determines theory, on the basis of constraints on theoretical construction.

Eschewing mathematical objects or theorems as starting points, Quine adopts a restrictive foundationalism. The alternative is transcendental. We start with substantive claims in epistemology and metaphysics, and seek a reflective equilibrium. Quine has already adopted this methodology in empirical science, by starting with objects. In the philosophy of language, one may lead with semantic claims, about truth or reference. The role for epistemology would then be to generate a plausible account of the links between the semantic claims and our abilities to learn and use language. Similarly, we should accept basic mathematical facts and look for good explanations of our knowledge of these.

Kant's attempt to generate epistemology transcendentally led him to make unjustifiable claims, both about our psychology and about the certainty of our knowledge. Neither of those are essential to a transcendental approach. Accepting some mathematical claims to begin an account of mathematical knowledge does not mean that one has to claim certainty for those claims. Once we sever the link between apriority and necessity, the transcendental approach is even more attractive. We can err about a necessary truth, without taking either the original proposition or its flaw to be held a posteriori. The transcendental approach does not necessarily countenance a priori knowledge, although it can easily accommodate apriorism. Even a naturalist can adopt a transcendental approach.[13]

The foundationalist will demand to know how it could be possible to settle questions about the existence of mathematical objects before the epistemic issues are settled. She worries that leading with metaphysics encourages untoward speculation. Errors have arisen this way: Kant's claim that Euclidean geometry is a necessary aspect of our psychology, Cartesian theism and dualism, Plato's denigration of the sensible world. This is a serious worry. The transcendentalist has to have at least a working hypothesis as to the nature of human epistemic capabilities.

Historically, the foundationalist is on equally tenuous footing. Hume's epistemology led to extreme skepticism, and Goodman's elaboration of the problems of induction show just how intractable those problems are, for the empiricist. All sorts of claims have been made about our epistemic

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[13] This is Devitt 1984's strategy for defending scientific realism. Also see Devitt 2003.

capabilities, in the absence of a clear and complete understanding of psychological and physiological data. Somehow, we perform ordinary inductions. Somehow, we have knowledge of ordinary objects around us. Somehow, we have mathematical knowledge. The job for the epistemologist, in conjunction with the empirical scientist, is to account for these abilities.

Devitt makes a similar point in response to sense-data reductionism. He notes that we can either take sense data to be mental, which leads to idealism, or non-mental, which leads to another sort of anti-realism. "Thus the price for knowledge turns out to be the abandonment of familiar reality. Epistemology determines metaphysics" (Devitt 1984: 70.)

The same criticism should be levied against Quine for his omission of mathematical objects as starting points. By requiring that only empirical objects require an account, Quine weakens his defense of mathematics. If calling this foundationalism seems a stretch, it is no matter. The point remains that Quine started with ordinary objects because he recognized that reductionism failed, but when he adopted the common objects of human knowledge, he should have included mathematical ones.

We should start our transcendental theorizing with a full mathematical ontology, including sets, numbers, and abstract spaces. Our specific commitments may be unclear for at least three reasons. First, mathematicians discover new theorems and generate new proofs which contain existential assertions. Second, debates over foundations, especially in set theory, have not generated universal agreement on the extent of the set-theoretic universe. Third, and related, depending on what we take as a criterion for mathematical existence, we may expand the mathematical universe or contract it significantly.

In Part 1 of this paper, I have argued that mathematics is a legitimate discipline independent of empirical science and that we should pursue a transcendental approach to epistemology and metaphysics, seeking a reflective equilibrium between the claims of mathematics and our epistemic abilities. In Part 2, I sketch the elements of this autonomy realism.

Part 2: Autonomy Realism

A detailed development of autonomy realism is beyond the scope of this paper. Still, a brief elaboration is warranted. In this second part of this paper, I first examine one autonomy realist theory, Balaguer's FBP, and find it wanting. Then, I present some of the central elements of any acceptable autonomy realism, including appeals to a fallibilist, *a priori* mathematical intuition, and the claim that many mathematical statements are necessary truths.

§2.1: Problems with FBP

One version of autonomy realism is Balaguer's plenitudinous platonism, or FBP, which claims that every consistent set of mathematical axioms truly describes a universe of mathematical objects.[14] For example, ZF + CH and ZF + not-CH each describe set-theoretic universes, despite their conflicting claims. In this section, I argue that FBP is not the best form of autonomy realism, because it generates too many objects in too many true theories.

FBP was inspired by Hartry Field's fictionalism, which I discuss in Part 3 of this paper. On Field's view, "[M]athematicians are free to search out interesting axioms, explore their consistency and their consequences, find more beauty in some than in others, choose certain sets of axioms for certain purposes and other conflicting sets for other purposes, and so forth; and they can dismiss questions about which axiom sets are *true* as bad philosophy" (Field 1998a: 320.)

FBP accepts all of these claims, except the last. While Field thinks that the mathematician is free because all his theories are false or vacuous, Balaguer thinks that the mathematician is free since all his theories are true. In either case, truth is no constraint on the construction of a mathematical theory.

Certain mathematical questions, like the question of the size of the continuum, have many conflicting answers where some people's intuitions say that there should be a unique answer. Gödel

[14] See Balaguer 1995 and Balaguer 1998: Chapters 3-4. Balaguer originally called his position full blooded platonism, hence the acronym.

famously claimed that the continuum has a unique size. But developments in the last few decades, especially Cohen's model-theoretic proof of the independence of the continuum hypothesis from the standard axioms of set theory, have undermined Gödel's claim, and those of anyone who claims that there is a unique size of the continuum. Various different and conflicting claims about the size of the continuum are all consistent with standard set theory. One could adopt stronger axioms which settle the question univocally. But, it seems to many set theorists that no unique answer is warranted.

One obvious solution to the problem of understanding the relations among the various set theories which result from adding different claims about the size of the continuum to the other axioms of set theory is to perform a Euclidean rescue: there can be several set-theoretic universes, each with their own defining set of axioms, which taken together are inconsistent. The traditional platonist, like Gödel, approaches such Euclidean rescues warily, preferring to find a unique answer. But, the fictionalist and the plenitudinous platonist embrace the results. The fictionalist says that statements of the different sizes of the continuum do not conflict because none of them are true. The plenitudinous platonist says that there are diverse set-theoretic universes.

But FBP provides no satisfactory account of our focus on preferred theories and interpretations, as on the standard model of the Peano postulates. "FBP-ists maintain that all consistent purely mathematical theories truly describe some collection of mathematical objects, but they do not claim that all such theories are true in a standard model" (Balaguer 1998: 60.) Every model of the axioms, on FBP, is equally acceptable. While focus on the standard model might be explained sociologically, and by its ubiquity, we may find gradations among different consistent theories. Some theories may be better mathematical theories than others.

Distinguishing between consistency and goodness is the main problem facing the autonomy realist. FBP only requires apprehension of consistency, not a contentious mathematical intuition. But it avoids the hard question of why we privilege certain systems. Justifying our interest in a standard model, if it is not to rest on application, will require appeal to intuition.

§2.2: Mathematical Intuition

The general form of my argument for autonomy realism is:

AR
- AR.1: We have mathematical knowledge.
- AR.2: Our mathematical knowledge must either be strictly derived from our scientific theories, or it must be autonomous.
- AR.3: Our mathematical knowledge is not strictly derived from our scientific theories.
- AR.C: Thus, autonomy realism.

AR.3 is established by the failure of the indispensability argument. AR.2 is uncontroversial. AR.1 remains a worry.

FBP attempts to account for mathematical knowledge on the basis of our pre-theoretic apprehension of consistency. The autonomy realist who wishes to explain our focus on the standard model will hone this criterion.

> One *might* adopt the ontological position that there are multiple 'universes of sets' and hold that nevertheless we have somehow mentally singled out one such universe of sets, even though anything we say that is true of it will be true of many others as well. But since it is totally obscure how we could have mentally singled out one such universe, I take it that this is not an option any plenitudinous platonist would want to pursue (Field 1998b: 335.)

On the contrary, this is exactly the position I have been pursuing. The obscure mental process to which Field refers is mathematical intuition. Any account of our knowledge of science must refer to our ability to reason about our commonsense beliefs. This ability to reason can not plausibly be limited to our knowledge of formal logic, since we must have some basis on which to develop such theories. One aspect of the evidence we have for science, for mathematics, and for logical theories must be our ability to reason. This ability is where the autonomy realist must look for accounts of mathematical intuition.

Field pointedly rejects intuition. "Someone *could* try to explain the reliability of these initially plausible mathematical judgments by saying that we have a special faculty of mathematical intuition that allows us direct access to the mathematical realm. I take it though that this is a desperate move..." (Field

1989a: 28.)[15]

Part of the worry about autonomy realism which makes it seem desperate is that it allegedly relies on a mysterious psychic ability. "The naturalism driving contemporary epistemology and cognitive psychology demands that we not settle for an account of mathematical knowledge based on processes, such as a priori intuition, that do not seem to be capable of scientific investigation or explanation" (Resnik 1997: 3-4.)

We must naturalize epistemology by debarring mystical and mysterious elements. Mathematical intuition must be compatible with a mature psychology. An epistemology which includes intuition is more than merely plausible. We reason all the time, in mundane matters as well as mathematics, logic, and linguistics. We can not rule out the possibility of a scientific, naturalistic explanation of our ubiquitous ability to reason. Charles Parsons, for example, assimilates mathematical intuition with our ordinary ability to identify types. "At least one type of essentially mathematical intuition , of symbol- and expression-types, is perfectly ordinary and recognized as such by ordinary language" (Parsons 1980: 155.)

Naturalism has become a dear doctrine to many philosophers as a way to avoid both mysticism and an unsatisfying empiricism. On the empiricism side, one is faced with the failures of logicism and positivism to provide uncontroversial justifications of mathematical knowledge. No one wants to return to a pure Millian account of mathematics.

On the mysticism side, we see accounts of mathematics from Plato, Descartes, and Gödel. Consider Putnam's remarks about Gödel's platonism. "The trouble with this sort of Platonism is that it seems flatly incompatible with the simple fact that we think with our brains, and not with immaterial souls. Gödel would reject this 'simple fact', as I just described it, as a mere naturalistic prejudice on my

[15] See also Field 1989a: 21, where he derides the sui generis solution to the Benacerraf puzzle about identifying numbers with sets; Field 1982a: 66-67, on Gödel's view, which Field misleadingly calls indispensabilist; and Field 1985b: 190.

part; but this seems to me to be rank medievalism on *his* part" (Putnam 1994: 503.)

If naturalism is to do any work at all, it must forestall an autonomous mathematical epistemology, as long as that is seen as entailing mysticism. Also, every one wants to avoid Kantian psychologism, which is like mysticism in positing substantial mental structures without empirical evidence.

One reason why one might think that autonomy realism requires mysticism is if one makes unreasonable demands on what counts as a non-mystical (i.e. naturalist) mathematical epistemology. For example, Putnam thinks that the autonomy realist requires a dedicated brain structure for mathematical perception. "We cannot envisage *any* kind of neural process that could even correspond to the 'perception of a mathematical object'" (Putnam 1994: 503.) Further, Putnam writes that appeals to intuition are, "[U]nhelpful as epistemology and unpersuasive as science. What neural process, after all, can be described as the perception of a mathematical object? Why of one mathematical object rather than another?" (Putnam 1980: 10.)

Any account of mathematical reasoning must be consistent with neuroscience, but this connection may be many degrees more subtle than the discovery of a region of the brain dedicated to mathematical perception. Putnam's demand for the details of neural processes which account for our apprehension of mathematical objects is too stringent. Indeed, the claim that there are neural processes which provide mathematical perception would be part of an empirical account of mathematics.

I do not pretend to have a neuroscientific account of mathematical intuition. But even a professed naturalist like Putnam recognizes the utility of appeals to intuition, though he grounds them unhelpfully in empirical science.[16] The alternatives to autonomy realism are too unsatisfying. Lacking full accounts of hard neuroscientific issues like consciousness, let alone apparently easier ones like perception, dismissing autonomy realism is too hasty, considering both the robustness of pure mathematics and the need for intuition to account for it.

[16] See Putnam 1994: 506.

In exactly the same way in which our knowledge of the external, physical world begins with an apprehension of physical objects in the absence of any plausible reductionist account to sensory stimulation, we begin our exploration of mathematical knowledge in the absence of an account of intuition. In both cases, the account must be judged on the basis of what is the best overall account of our common-sense views of both mathematics and the physical world. It may turn out that the best account of our purported mathematical knowledge involves denying that we have any. But such a denial must carry with it an account of why we seem to have mathematical knowledge, and this is a task which no eliminativist has accomplished.

Mathematical intuition, or, less contentiously, mathematical reasoning, allows us to distinguish among consistent mathematical theories. It will, of course, be an a priori method of belief formation. A traditional objection to intuition relies on the impossibility of forming substantial, yet indefeasible, mathematical beliefs. This objection does not affect the fallibilist a priori I have touted.

The autonomy realist requires an a priori faculty of mathematical intuition. The account of this faculty will include a detailed examination of all our belief-forming processes, and a partition of these into a priori and empirical classes. I have not provided such an account in this dissertation. Relatedly, autonomy realism should include an account of mathematical necessity, which I describe in the next section.

## §2.3: Necessity

For the indispensabilist, mathematical objects exist as contingently as the physical world. This Unfortunate Consequence pushes us toward autonomy realism. But if we adopt FBP as our autonomy realism, then, according to Balaguer, we still suffer from Modal Uniformity. In this section, I argue that this is another reason to reject FBP, and to seek a more refined autonomy realism.

Balaguer bases his rejection of necessity on its obscurity. Mathematical existence claims are not logically or conceptually true, he argues. Adopting Field's stance, he says there is no other coherent view of necessity.

Balaguer argues that the realist only employs necessity to ground his epistemology, as Jerrold Katz and David Lewis do. They argue that since mathematical objects exist necessarily, there is no need to account for the conditions of their existence.

> There problem here is that we just don't have any well-motivated account of what metaphysical necessity consists in. Now, I suppose that Katz-Lewis platonists *might* be able to cook up an intuitively pleasing definition that clearly entails that the existence claims of mathematics - and, indeed, all purely mathematical truths - are metaphysically necessary. If they could do this, then their claim that mathematical truths are necessary would be innocuous after all. But (a)...the claim would still be epistemologically useless, and (b) it seems highly unlikely (to me, anyway) that Katz-Lewis platonists could really produce an adequate definition of metaphysical necessity. (Balaguer 1998: 44-45.)

Claim (a) might be right, though it is not far from Balaguer's own claim. FBP says that every consistent mathematical theory truly describes a mathematical universe. This is very close to saying that the theorems of mathematics, when true, are necessarily true, and that mathematical objects exist necessarily. Moreover, the necessity of mathematics can help explain why consistency entails truth.

Balaguer's real argument here is (b). Balaguer and Field are right that mathematical necessity can not be logical necessity. But I am not as skeptical as Balaguer about the prospects for an account of necessity for mathematics, and I do not see his argument against it.

There are at least two reasons to want an account of mathematics on which mathematical objects exist necessarily. Besides the Katz-Lewis view, which uses necessity to ground an epistemology, one might merely wish to account for the common belief that there is a difference between what might have been different and what could not have been different. Autonomy realism need not concede necessity.

Given the failure of indispensability, philosophers of mathematics really have two choices: either deny that mathematical objects exist, or show how we can know of them independently of science. The most important opponent of the autonomy realist, then, is the anti-realist. In the final part of this chapter, I show that autonomy realism compares favorably to the most plausible anti-realism, Field's fictionalism.

Part 3: Autonomy Realism and Fictionalism

In this last part of the paper, I discuss three problems with Field's fictionalism. It provides a weakened account of the difference between mathematical truth and falsity. It unavoidably assimilates mathematical statements to ones about which we get to say whatever we like. And Field defends fictionalism, which denies that mathematical objects exist, but he really only generates skepticism.

Normally, we distinguish between '2+3=5' and '2+3=6' by calling the former true and the latter false. If we call all statements which refer to mathematical objects false, then both of these claims are equally false.[17] The fictionalist requires that we distinguish the two sentences on the basis of applicability and utility. '2+3=5' is false, but useful, and conservativeness accounts for utility.

False mathematical theories may be useful, just as false scientific theories may be useful. If we distinguish what we normally call truth from what we normally call falsity based on utility and application, we assimilate mathematical truths to mathematical falsities.

The fictionalist denies that Wiles showed us something new about mathematical objects when he showed that there are no $n > 2$ for which $a^n + b^n = c^n$. We already knew that, since there are no numbers at all. Field will credit Wiles with advancing our logical knowledge, but this is a weakened account of what we learned. "I imagine most mathematicians would be contemptuous of this speech and most philosophers - even most *nominalist* philosophers - embarrassed by it" (Burgess 2004: 24).

In addition to the anemic account of mathematical progress, fictionalism wrongly assimilates mathematical sentences to other fictions which lack constraints about what we say concerning them. Fiction can defy physical and even mathematical possibility. In contrast, we do not have full freedom to say whatever we like in mathematics. A mathematical theory must at least be consistent. We seek interesting problems in mathematics, but even uninteresting problems and solutions can be

[17] I take these simple sentences as meaning that there are numbers two, three, and five, and the sum of two and three is five, and similarly for the second. If one takes them as entailments from Peano axioms, then the problem is that the former is vacuously true when it should be true because of the relations among 2, 3, and 5.

mathematically good. There is nothing mathematically wrong with demonstrating, in set-theoretic language, the sum of 43 and 171, as long as we get 214, in the way that there would be something wrong if we were to conclude that the answer is seven billion.

The fictionalist may respond that there are constraints on non-mathematical fictions, too, depending on how we take the metaphor between mathematics and fiction. Burgess considers a variety of options. Is mathematics like novels? If so, then we really should have full freedom to create it in whatever way we please; the metaphor fails. Or is mathematics like mythology, as Leslie Tharp suggested? Or metaphors, as Stephen Yablo suggested? Or fables? If we take fables or mythology as paradigms, we may defend a constraint on mathematics, derivative from the constraints on mythology and on fables. We can not make Athena the goddess of grain.

The fictionalist would be unwise to assimilate mathematics to myths or fables. Aligning the accounts of mathematical goodness and with mythology would lead to worrisome questions about the standards for establishing mathematical theorems. Myths are hardly evaluated at all. We can construct new myths, but these need not be consistent with the old myths.

Field argues that good mathematics is conservative, and conservativeness is close to consistency.[18] The fictionalist thus has standards for distinguishing among theories. But the account of mathematical conservativeness is not as clean as Field would like. The fictionalist cares about consistency only as a pragmatic condition on conservativeness. An inconsistent mathematical theory is no longer conservative, implying new nominalistically acceptable conclusions.

The difference between our freedom to construct fiction and the constraints on mathematics is not

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[18] Conservativeness is the claim that the addition of mathematics to the reformulated theory should license no additional nominalist conclusions. Here is Field's formal definition: Let A be any nominalistically statable assertion, N any body of such assertions, and S any mathematical theory. Take 'Mx' to mean that x is a mathematical object. Let A* and N* be restatements of A and N with a restriction of the quantifiers to non-mathematical objects. This restriction yields an agnostic version of the nominalist theory; it does not rule out the existence of mathematical objects. S is conservative over N* if A* is not a consequence of N*+S+'$\exists x \sim Mx$' unless A is a consequence of N. '$\exists x \sim Mx$', that there is at least one non-mathematical object, is a technical convenience. See Field 1980: 10-16.

decisive against the fictionalist. The fictionalist need not commit to a positive account of mathematics based on the positive account of novels, say. Field does not suggest that we abandon our standard mathematical criteria for acceptance of theorems, or revolutionize mathematical practice. But by making the analogy with fiction, he invites such comparisons.

The problems of calling mathematical claims false and assimilating mathematics to fiction apply to any fictionalism. An additional problem arises for Field's particular version. Field grants that mathematical objects could have existed. This is a result of his object-level modal operator.[19] The possible existence of mathematical objects leads to skepticism about mathematical objects, not fictionalism.

Consider the world as it is, and accept with the fictionalist the contingent non-existence of abstract objects. Now, imagine that numbers are suddenly created.[20] Field's modal account renders this possible. By causal isolation of abstract objects, we are in principle unable to know of them. The fictionalist can not say that abstracta do not exist, but only that we have no way of knowing.

This problem does not arise for the ordinary nominalist who relies on considerations about our isolation from mathematical objects to deny their existence. The problem arises for Field because he conjoins fictionalism with an account of consistency which makes mathematical objects possible, but not actual.

None of the three problems I discussed in this section rely on taking mathematics to be indispensable to science, as other arguments against fictionalism do. For example, Maddy argues that fictionalism fails to account for why one mathematical story seems most important. "Oliver Twist, whatever his other virtues, lives only in one good story among others, but the characters of mathematics,

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[19] Field represents our knowledge of the consistency of the axioms of ZF as'$\Diamond AX_{ZF}$'. See Field 1984.

[20] Azzouni argues that nothing would change if mathematical objects ceased to exist, in Azzouni 1994: 56. David Lewis denies that we can say anything sensible about how the world would be if there were no numbers (Lewis 1986: 111).

no matter how we twist and turn, have a stubborn way of introducing themselves into a story that is our very best of all" (Maddy 1990b: 204.)

Maddy thus denies fictionalism on indispensabilist grounds. There is a best theory, she says, and we can not seem to avoid mathematics when formulating it. If we had to decide between fictionalism and indispensabilism on this basis, the fictionalist theory is preferable. The fictionalist can see the role of mathematics in a best theory as a pragmatic matter. Different mathematical stories apply differently to different physical worlds. But the fictionalist leaves us without an account of the ubiquity of mathematics to which Maddy alludes.

The autonomy realist, in fact, has a better account of application, since it is broader. The autonomy realism is best able to account for new applications of mathematics since it provides whatever mathematics might ever be needed in science.

Field argues that he can best account for the application of mathematics to empirical science, through appeal to conservativeness. As evidence, he cites Michael Friedman as having rejected Field's nominalism, while, "[E]ndorsing its account of the applications of mathematics" (Field 1985a: 191).[21]

Field recognizes, though, that the realist can also take his representation theorems as an account of application. The realist does not deny the legitimacy of the fictionalist's tools; he just has more. Yet on his own terms, Field does not explain application, since the representation theorems are not available in the official, first-order version of his theory. If he can develop representation theorems for all applications of mathematics, then the realist and the fictionalist can use the same account.

The autonomy realist, ex hypothesi, has an epistemology for mathematical objects as well as the independent account for empirical objects. For FBP, for example, our knowledge of mathematical objects arises from our ability to recognize contradictions, or non-contradictory sets of theorems. FBP entails a mathematical description for every possible empirical situation. Thus, FBP can provide an additional

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[21] Friedman actually says, "There is no doubt that it is a major *contribution* to our understanding of applied mathematics" (Friedman 1982: 506), which need not be interpreted as endorsing the account.

account of the applicability of mathematical objects, in case the fictionalist's account fails.

Fictionalism denies any account of the axioms forcing themselves on us, beyond the applicability of mathematics. Gödel took the feeling of constraint to be evidence of mathematical intuition. "[D]espite their remoteness from sense experience, we do have something like a perception also of the objects of set theory, as is seen from the fact that the axioms force themselves upon us as being true." (Gödel 1963: 483-484; see also Putnam 1994: 503, quoting Hao Wang)

The feeling of discovering mathematical truths is not really an argument. The fictionalist can urge us to think of this feeling as of something else, like having discovered a logical entailment. But it adds to the amount of re-thinking that one must do to embrace fictionalism, and this coheres poorly with Field's claim that we are to take mathematics at face value.

The fictionalist denies that mathematical theories have a subject matter. Consider the problem of how to interpret conflicting consistent set theories, like ZF with the axiom of choice and ZF with its negation. The indispensabilist allows only one universe of sets, that which applies in science. Either there is a choice set for any set or there is not.

Fictionalism avoids this difficulty: there is no choice set. "Our different set theories 'have a different subject matter' only in that they are different stories. They differ in subject matter in the way that *Catch 22* and *Portnoy's Complaint* differ in subject matter; these differ in subject matter despite the fact that neither has a real subject matter at all...[N]either is properly evaluated in terms of how well it describes a real subject matter." (Field 1990: 207)

The autonomy realist can also take this stance, accepting multiple set-theoretic universes, each with different objects, each with different theorems true of it. The problems with Field's fictionalism lead us to autonomy realism, just as the problems with indispensability did.

Burgess and Rosen understand as well as any philosophers the limits of the indispensability argument and the motivations for autonomy realism, but they resist what I take to be the obvious conclusion. They take naturalism and its descriptive methodology as a sufficient to reject a priori, or "alienated" epistemology. "The two stand to each other rather as the descriptive grammar of Chomsky stands to the prescriptive grammar of Fowler" (Burgess and Rosen 1997: 209.) They rule out dispensabilist reformulations on the basis of their unattractiveness. They reject fictionalism, too, leaving only indispensability as an option for an account of mathematics. They find indispensability unacceptable on the grounds that parsimony is not a scientific merit. They have shown all possible positions untenable.

The lesson for Burgess and Rosen, and for all of us, is to reconsider which elements of so-called alienated epistemology might be acceptable. We should pursue autonomy realism.